Probabilistic Consideration on Chances of Invasion between the States

-- The Symmetry --

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Japan's Constitution outlaws any armament, though not small armed forces and weapons already exist. There are constantly debates to change the Constitution to legitimate this army and even to send it abroad.
Hawks attacks the defenders of the Constitution by putting a question:

> "What can you do without army when our country is invaded by another countries?"

Though it is a natural question and has to be answered anyhow, it should be reminded that the following question must be equally raised:

> "What can we do when our army invades another countries?"

These possibilities are mathematically equal, if one assumes no a priori difference in the nature of countries --agressive or peaceful.
The following is not a realistic discussion but only a simple mathematical one. Nevertheless I believe it will help one to remind that these two questions should be posed equally. (The first question is not a realistc one because Japan keeps a large-scale army now.)

One might think that the probability of being invaded is greater because the number of potential invader to his/her country is plenty but his/her country which may invade is unique. If you don't think so, you need not read hereafter.

Suppose we have n+1 states whose probabilities of invasion to other states are all equal.
Let p be this probability of invasion by **a** state to **any** other states in a certain period of time. The quantity p is also an expected value of the **frequency** of invasion by a state in this period. Then the expected frequency of invasion by a state to **one** of other states is p/n. We assume no correlation among the behaviour of different countries, i.e., no alliance exists.

Let us calculate the expected frequency of invasion to **a** state by **any** other states.
The probability $P(k)$ of being invaded **simultaneously** by k states is

$$P(k)=\left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

and its number of cases is ${}_{n}\mathrm{C}_{k}$.
Then the total expectation of **frequency** of invasion by **all** of other states is obtained by summing up all possible values of k weighted by ${}_{n}\mathrm{C}_{k}\,P(k)$:

$$\langle f\rangle=\sum_{k=1}^{n}k\;{}_{n}\mathrm{C}_{k}\;P(k)=\sum_{k=1}^{n}k\;{}_{n}\mathrm{C}_{k}\left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

$$=\sum_{k=1}^{n}\frac{n\,!}{(n-k)\,!\,(k-1)\,!}\left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

Replacing k with $r+1$, this expression becomes

$$\langle f\rangle=\sum_{r=0}^{n-1}\frac{n\,!}{(n-r-1)\,!\,(r)\,!}\left(\frac{p}{n}\right)^{r+1}\left(1-\frac{p}{n}\right)^{n-r-1}$$

$$=n\cdot\frac{p}{n}\sum_{r=0}^{n-1}\frac{(n-1)\,!}{(n-r-1)\,!\,(r)\,!}\left(\frac{p}{n}\right)^{r}\left(1-\frac{p}{n}\right)^{n-1-r}$$

$$=n\cdot\frac{p}{n}\left(\frac{p}{n}+1-\frac{p}{n}\right)^{n-1}$$

$$=p$$

Namely, $\langle f\rangle$ is simply equal to p.

This result is an anticipated one because "defensive war" and "aggression" are only different names for a same event, the war.

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