Proposition of a nanoscale collider experiment to examine the loss of visibility due to entanglement

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Monochromatic and point-like source which emits a particle beam without wave-like property is proposed. Such a source can be realized by taking halves of entangled pairs. Experimental feasibility to realize such a source is discussed in detail.

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1 Introduction

The matter wave has a coherence property, like classical waves, if the source size is small and the spread of the beam energy is narrow. The degree of coherence is examined by observing interference fringes. However, the particle beam does not show the interference property if each particle undergoes an interaction with another particle before entering into an interferometer. It does not matter however small the size of this interaction region or the energy spread of the emerging particles is.

This is due to the fact that they make an entangled pair [1]. (This phenomenon is also studied by some authors and are called "decoherence" [2] or "quantum-mechanical phase relaxation" [3].) If we observe a half of an entangled pair, we see that the interference property as a single particle is lost, even though the source is monochromatic and pointlike. Thus we obtain a very exotic particle source which shows no apparent wavelike behavior in spite of its high spatial and temporal "coherence".

Since there seems to be no experimental demonstration of such a particle source, it is worthwhile to realize it. We will show that this can be done with existing technologies.

2 Loss of wave-like property of a particle due to entanglement

We can understand this effect as that an integration over the coordinate of the non-observed particle of the entangled pair erases the interference term. Suppose particles 1, 2 in states $|a\rangle_1, |\bar{a}\rangle_2, |b\rangle_1$, and $|\bar{b}\rangle_2$ are coupled as

$$|\Psi\rangle = |a\rangle_1|\bar{a}\rangle_2 + |b\rangle_1|\bar{b}\rangle_2\,.$$

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Then, if we observe only the particle 1, we must calculate $|\langle \boldsymbol{r}_1|\Psi\rangle|^2$ by integrating over the position of the particle 2.

$$|\langle \boldsymbol{r}_1|\Psi\rangle|^2 = |\phi_a(\boldsymbol{r}_1)|^2\langle\bar{a}|\bar{a}\rangle_2 + |\phi_b(\boldsymbol{r}_1)|^2\langle\bar{b}|\bar{b}\rangle_2 + 2\,\mathrm{Re}[\phi_a^*(\boldsymbol{r}_1)\phi_b(\boldsymbol{r}_1)\langle\bar{a}|\bar{b}\rangle_2]\,, \quad (1)$$

where ϕ_a and ϕ_b are for $\langle \boldsymbol{r}_1|a\rangle$ and $\langle \boldsymbol{r}_1|b\rangle$. We assume orthonormality of the wavefunction of the particle 2,

$$\langle\bar{a}|\bar{a}\rangle_2 = \langle\bar{b}|\bar{b}\rangle_2 = 1 \quad \text{and} \quad \langle\bar{a}|\bar{b}\rangle_2 = 0\,,$$

so that $|\langle \boldsymbol{r}_1|\Psi\rangle|^2 = |\phi_a(\boldsymbol{r}_1)|^2 + |\phi_b(\boldsymbol{r}_1)|^2$, leaving no interference term. Inclusion of the symmetry property of the wavefunction due to indistinguishability of colliding particles does not alter this conclusion.

Such a pair can be prepared as a momentum entangled pair of electrons simply by an elastic collision, as shown in Fig. 1. If we are nearly on the center-of-mass system and the collision region can be limited to a very small volume, then we will have a very strange particle source which have a temporal and spatial "coherence" but shows no interference effect at all. Since the spin-flip probability is negligible at small collision energy, inclusion of the spin wavefuncition does not affect the above discussion.

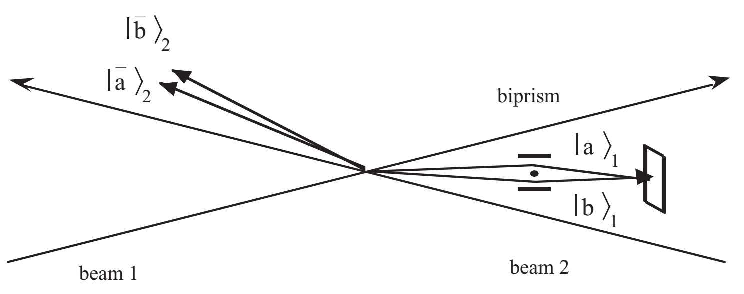

Fig. 1. Proposed experimental arrangement. Electrons mutually scattered from the beam crossing region are put through the interferometer.

It is essential that the loss of interference property is not due to the trivial reasons, namely the size of the source and the non-monochromatic nature of the beam. We will discuss how to prepare such a source in detail for electrons.

3 Experimental feasibility to obtain an electron source which emits non-wavelike beams

An electron source with such a property can be realized by colliding thin electron beams of the same energy at a nearly head-on geometry. If the beams are collided exactly head-on, we have no variation in energy over the scattering angles. However the collision volume cannot be defined with DC beams at this geometry. So we make cross two beams at several degrees less than 180 degrees.

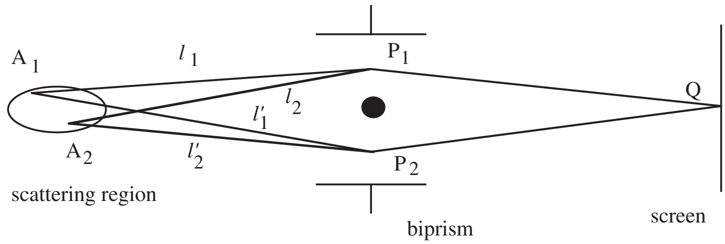

Fig. 2. Definition of parameters of the interferometer.

Main origin of the energy spread of the mutually scattered particles is the spread of the angles of incoming beams. To minimize this effect, both beams are truncated within small solid angles, and the scattering (observation) angle as close as possible to both beam axes is taken. Moelenstedt biprism is used as a sensitive interferometer. Path lengths and their differences are denoted as follows (see Fig. 2):

$$l_1 = \overline{\mathrm{A_1P_1Q}}\,,\ l_1' = \overline{\mathrm{A_1P_2Q}}\,,\ l_2 = \overline{\mathrm{A_2P_1Q}}\,,\ l_2' = \overline{\mathrm{A_2P_2Q}}\,,$$

$$\Delta l_1 = |l_1 - l_1'|\,,\ \Delta l_2 = |l_2 - l_2'|\,,\ \Delta(\Delta l) = |\Delta l_1 - \Delta l_2|\,.$$

We must be sure that the loss of visibility or interference is neither because of the spreads of energy nor because of the size of the source. Therefore $\Delta l(\Delta l_1$ or $\Delta l_2)$ should be smaller than the longitudinal coherence length, and $\Delta(\Delta l)$ should be small so that the particles originated from different points of the source (A_1 and A_2 in Fig. 2) may undergo nearly the same phase differences between the two paths around the anode wire of the biprism. These requirements are expressed as follows:

$$\Delta k\,\Delta l \ll \pi\,, \tag{2}$$

$$k\,\Delta(\Delta l) \ll \pi\,, \tag{3}$$

where k and Δk are the wave number and its spread of the scattered electrons.

Denoting the size of the biprism ($\overline{\mathrm{P_1P_2}}$) by d, the distance from the source to the biprism by L and the size of the scattering region perpendicular to the direction of observation (to the biprism) by $s_{\max}$, then Δl is approximated by $s_{\max}d/2L$. Since we choose small crossing angle of two beams ($\beta_1 + \beta_2$ in Fig. 3) and the biprism is placed close to the both beam lines, then $s_{\max} \approx W$, where W is the width of the beam. Therefore we have $\Delta l \approx Wd/(2L)$. $\Delta(\Delta l)$ is equal to Δl and we obtain a usual relation for the lateral coherence: $kWd/(2L) \ll \pi$.

Now we are ready for a numerical evaluation. We assume the beam energy to be 2.5 keV ($k = 2.56 \times 10^{11}\,\mathrm{m}^{-1}$) and the width of the beam to be 5 nm. We suppose the angular spread can be limited to less than 0.02 rad (1.15 degree) by strongly collimating the beam. We take 3.5 degrees for β_1 and β_2. With this geometry, kinematical energy spreads due to the spreads of angles of the colliding electrons are 9.1 eV and 2.96 eV for $\Delta\beta_1 = \Delta\beta_2 = 0.02$ rad, respectively. For the biprism, we assume $L = 100$ mm and $d = 0.1$ mm.

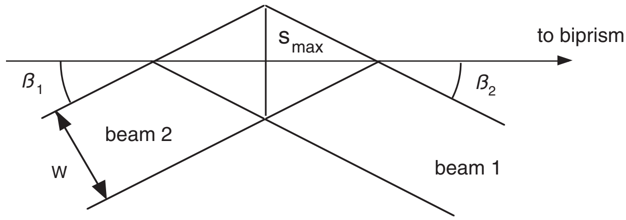

Fig. 3. Definition of parameters for the collision region.

With these parameters the quantities of (2) and (3) are estimated as follows:

$$\Delta k\,\Delta l = 1.17\times 10^{-4}\,,\quad k\,\Delta(\Delta l) = 0.64\,.$$

Although the requirement of fulfilling the lateral coherence is rather strong, it can be within our technical effort.

4 Do the angular spreads of the incident beams blur the "which-path" information?

Above we have seen that the energy broadening of the scattered particles due to the angular spread of the incident beam is negligible. However, it also has the effect that the "which-path" information could be blurred and may eventually recover the interference fringe. Supposed source size of 5nm is sufficiently small so that it can be considered to be point-like for the biprism interferometer ($W \ll 2\pi L/(kd)$, right hand side is 24.5 nm). The uncertainty principle defines the minimum angular spread of the incident beam to focus to a small collision region W by the inequality $\Delta p_\perp W \geq h$, where $\Delta p_\perp$ denotes the momentum spread perpendicular to the beam direction. This spread is copied to the angular spread of the scattered particle.

If we take x-axis from the collision point to the biprism and y-axis perpendicular to this and to the biprism filament, the uncertainty principle reads as $\Delta(p_{y1} + p_{y2})\,\Delta(y_1 + y_2) \geq h$, where p_{y1} and p_{y2} are the components of momenta of the incident particle 1 and 2, respectively, and y_1, y_2 are their positions. The same inequality holds for scattered particles and if the properties of two beams are similar, it also holds for the particles going to the biprism, namely $\Delta p'_{y1}\Delta y_1 \geq h$.

Momentum spread $\Delta p'_{y1}$ is expressed as $\hbar k\Delta\theta$ where $\Delta\theta$ is the angular spread of the scattered particles. Using $\Delta\theta$ and replacing Δy_1 by W, the above inequality is expressed as

$$\Delta\theta \geq \frac{2\pi}{kW}\,. \tag{4}$$

With the experimental conditions described here, the right-hand side amounts to 4.9×10^{-3}, which is 5 times greater than the angular size of the interferometer viewed from the collision point. In fact, (4) is written as $W \geq 2\pi L/(kd)$ by replacing $\Delta\theta$ by d/L, and contradicts the condition that the source should be point-like. This could mean that, with the experimental condition proposed here, the interference fringe does not vanish in spite of the entanglement. Then we must prepare the electron beams, which have angular spread less than d/L, focus size W becoming greater as much as allowed by the uncertainty principle. By this, we may be able to observe the lowering of the contrast of the fringe due to entanglement. However, if each incident beam is considered to be a statistical ensemble of particles having small angular spreads, disappearance of fringe may occur with beams of a large angular spread. All of the above questions are to be answered in the laboratory.

5 Yield and "calibration"

The luminosity of the collision region is roughly estimated to be

$$L = \frac{I^2}{e^2 W v \sin(2\beta)}, \tag{5}$$

where v is the velocity of the electron, I is the beam intensity in ampere and 2β is the beam crossing angle $\beta_1 + \beta_2$. For simplicity, we assumed a square cross section with an area of W^2 for each beam and thereby the volume of the collision region is $W^3/\sin(2\beta)$.

We assume a brightness of $10^8\,\mathrm{A/cm^2 sr}$ for each field emission source. If it is multiplied by our focus area of $2.5 \times 10^{-13}\,\mathrm{cm}^2$ and the accepted angular divergence of $3.1 \times 10^{-4}\,\mathrm{sr}$, we obtain a beam intensity of 7.9 nA. Inserting it and other parameters into (5), we have a luminosity of $1.3 \times 10^{23}\,\mathrm{m^{-2}s^{-1}}$. Using the Mott formula for the Coulomb scattering cross section, we have $5.89 \times 10^{-21}\,\mathrm{m^2/sr}$ for the scattering angle of 3.5 degrees and the center-of-mass energy of 4.98 keV. If we put the biprism with an aperture of $0.1\,\mathrm{mm} \times 5\,\mathrm{mm}$ at a distance of 100 mm from the collision region, then we can expect a counting rate of 0.04 counts/s or 2.4 counts/min.

It is essential to confirm that the lack of fringes is not due to the source size or the energy spread. Namely we must check experimentally that the apparatus is inherently capable of observing fringes. This is performed by preparing the same interferometer for the particle 2 and by taking coincidence with the particle 1. Here we will get non-zero values $\phi^*_{\bar{a}}(\boldsymbol{r}_2)\phi_{\bar{b}}(\boldsymbol{r}_2)$ in place of the spatial integration $\langle \bar{a}|\bar{b}\rangle_2$ in Eq. (1). After simple calculation we get $2\cos[|\Delta\boldsymbol{k}|(x_1 - x_2)]$ for the interference term in Eq. (1), where $\Delta\boldsymbol{k} = \boldsymbol{k}_a - \boldsymbol{k}_b = \boldsymbol{k}_{\bar{b}} - \boldsymbol{k}_{\bar{a}}$ and x_1, x_2 denote positions on the two observation planes along the direction of $\Delta\boldsymbol{k}$. Then, if we observe a correlation of two positions of electrons on the both planes, we will obtain a fringe pattern on the x_1–x_2 plane. For this purpose we could use a combination of an electron bombarded CCD (EB-CCD) [1]) with a microchannel plate as a position detector for low-energy electrons, after magnification by electron lenses. Since the expected counting rate is very low, continuous recording of a few pairs of images per second is sufficient. Otherwise we can use a position-insensitive detector with a mask whose aperture spacing is $2\pi/|\Delta\boldsymbol{k}|$ as a "trigger" detector for the position sensitive detector (MCP + CCD). With this combination we can expect a fringe pattern on the CCD. This

[1]) Response of the CCD to the low energy electrons is reported in [4].

method sacrifices a counting efficiency but we can avoid a huge amount of null data which is inevitable if we use two CCDs on both sides.

6 Possible extension to an experiment to decide the level where the wavefunction collapses

Bussey [5] proposed a 'thought' experiment to test if the wavefunction collapses at the level of microscopic events. An extension of our proposed setup may turn it a real experiment. Their proposal is to collide two particles and then combine the scattered and unscattered waves of each particle by a pair of Mach–Zehnder type interferometers (MZI). Different patterns in coincidence rates between the four outputs of the two MZIs are expected according to the collapse/non collapse assumption.

If we use biprism as two arms of MZI and place a beam splitter instead of the screen Q in Fig. 2, and providing the same for another scattered electron, we could realize the Bussey's experiment. For beam splitters we could use very thin sheets of single crystals, described by Marton *et al.* [6], which function as diffraction lattices. It is worth studying on the experimental feasibility of this setup.

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